Axiomatic Design Based Complexity Measures to Assess Product and Process Structures

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Abstract. Definitions of complexity often depend on several circumstances, such as the nature of investigated complex system, the kind of complexity, the conceptual framework used for a study, the theoretical approach taken, and the like. In this paper, two complexity measures that are based on Boltzmann's entropy concept and AD theory are proposed and described. The first measure quantifies product variety complexity and the second one focuses on process structure complexity. Such complexity techniques will be used to determine product platform complexity and related process complexity for early stage of design decision-making. The method focused on product platform complexity assumes that the distribution of FR-DP couplings offers a suitable complexity concept, which prescribes that coupled designs should be decoupled, if possible, since uncoupled design is ideal and a decoupled design is less good, while a coupled design is the least satisfactory. Analogically, the same principle is used for the purpose to quantify topological process complexity by transforming input components into process variables and product modules including final product into design parameters. Subsequently, relevant properties of these measures will be analysed by computational experiments. Finally, practical findings for mass customization practice will be presented.

1 Introduction

Definitions of complexity often depend on several circumstances, such as the nature of investigated complex system, the kind of complexity, the conceptual framework used for a study, the theoretical approach taken, and the like. Overviews of complexity approaches and theories were offered by number of studies, see, e.g., [1-4] which provide complementary information among them. According to Gao et al [5] Shannon information concept seems widely recognized as essential building block of complexity theory. Traditional complexity metrics are associated with an absolute measure of complexity in contrast to the axiomatic design complexity, where this system attribute is treated as relative quantity based on the information concept and axioms of Axiomatic Design (AD). The complexity is expressed as: "A measure of uncertainty in understanding what it is we want to know or in achieving a functional requirement (FR) [6]." A relative information quantity in this approach is determined by the overlap between the system range of FRs and the design range of FRs. By this complexity theory, four different types of complexities: time-independent real complexity, time independent imaginary complexity, time-dependent combinatorial complexity, and time dependent periodic complexity are identified. The key idea of the theory is reduction of a complexity of system design in order to increase system reliability at each level of the design hierarchy. The dependencies between the FRs and the design parameters (DPs) can be classified by three types of design matrix (DM): uncoupled, decoupled and coupled. Among them, coupled design where FRs are influenced by possible changes of individual FRs, is more complex than an uncoupled design. Guenov [7] extends AD complexity theory for computing the information content of all types of design matrices. For this purpose, he developed complexity measures of design representations by adopting Boltzmann's entropy concept. Existing research literature on AD (see, e.g. [8-11]) offers other challenging approaches and inspirational studies including complexity issues.

In this paper, two complexity measures are proposed. The first one, determines product variety complexity and the second one is dedicated to measure process structure complexity. Both of them adopt one of the complexity measures from the work by Guenov [7].

Such complexity measures will be used to define product platform complexity and related process complexity for early stage of design decision-making based on the assessment of topological structures of design matrices. The method focused on product platform complexity assumes that the distribution of FR-DP couplings offers a suitable complexity concept, which prescribes that coupled designs should be decoupled if possible, since uncoupled design is ideal and a decoupled design is less good, while a coupled design is the least satisfactory [6]. Analogically, the same principle is used for the purpose of quantifying static process complexity based by

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transforming input components into process variables and product modules including final product into design parameters.

2 Theoretical background

Adapted complexity measures used in our approach are based on Boltzmann's formula [12] for statistical entropy introduced in 1872. This formula in case of an ideal gas exactly corresponds to the thermodynamic entropy. According to Anderson [13] Boltzmann's entropy is simply related to disorder in many field of science, even though this resemblance for the most part have nothing to do with the second law of thermodynamics. Guenov [7] substituted number of elements (molecules) in formula for degree of disorder (Ω) by total number of couplings (N) in design matrix in order to calculate design complexity based on the assessment of topological structure of the design matrix as follows:

$$\Omega = N\,!/(N1\,!...N\,K\,!), \qquad (1)$$

where K is number of design parameters (number of columns in design matrix).
The same author recognized that formula:

$$SDC = \Sigma N_j \ln N_j, \qquad (2)$$

which was derived from the statistical entropy seems to convey better the meaning of axiom 1 of AD, where N_j is the number of couplings per design parameter (i.e. per column), $j = 1,..., K$.
Therefore, this complexity measure denoted by us as Systems Design Complexity (SDC), will be applied for the purpose to estimate *product variety complexity* and *process structure complexity*.
A simple example demonstrating the application of this measure can be shown by using random design matrix (DM) with its couplings distribution depicted in Figure 1.

	DP_1	DP_2	DP_3	DP_4	DP_5
FR_1	X	0	0	X	X
FR_2	0	X	0	0	0
FR_3	0	0	X	0	0
FR_4	0	0	0	X	0
FR_5	0	X	X	0	X

Fig. 1. An example of DM with couplings denoted by symbol "X".

Then, SDC= $\Sigma N_j \ln N_j$ = 1ln1 + 2ln2 + 2ln2 + 2ln2 + 2ln2 = 5,55 nats.

3 Adaptation of SDC to measurement of product variety complexity

3.1. Description of the product complexity measure

In order to adopt SDC measure for quantifying product variety complexity, the following steps are involved:

1) Classification of input components (ICs) entering an assembly process in terms of mass customization,

2) Description of graphical model interpreting relation between ICs and possible product (module) configurations (PPCs),

3) Conception of transformation mechanism of the relation between ICs and PPCs into a DM.

3.1.1. Classification of input components

Due to the fact the structure of variable product components determines total number of product combinations, it is reasonable to introduce working classification of input components. We consider three types of initial components entering an assembly process. They are as follows:

Stable components (S) are considered to be assembled for ensuring the functionality of the module or final product.

Optional components (O) are useful in some cases but they are not required. They can be selected according to the customer´s requirements and are optional in any combination, including cases when only individual components are chosen. Selection by customer without this type of component is also an option.

Compulsory optional components (CO) are different from O by the number of components that may be chosen from all of them. They are limited in selection. Thus, restrictions are determined by three specific selection rules: minimum, maximum and exact requirements on selection. These selection rules can be specified in a simple way by combinatorial number $\binom{k}{l}$, where l defines ways of picking component combinations from a set of all k, while $1 \leq l < k$.

In mass customization environment, practically any number of stable, voluntary and compulsory optional components can be combined. However, the following specific selection rules of the selections for the set of CO components with number k may occur when identifying product configurations. They are:

Individual selectivity rule, where it is possible to define exact number of 'l' of components to be chosen from all 'k' of CO components;

Maximum selectivity rule where it is possible to define the maximum number 'l' of CO components to combine within an assembly choice of all 'k' of CO components (note that 'l' is max. k-1);

Minimum selectivity rule where it is possible to choose/combine at least '*l*' CO of components from available '*k*' of CO components (note that '*l*' is min. *1*).

Combinatorial formulas for determining numbers of product (module) configurations according to the above described components types and selection rules are presented by Modrak [14]. The example of customizable chair containing of three types of initial components is shown in Fig. 2.

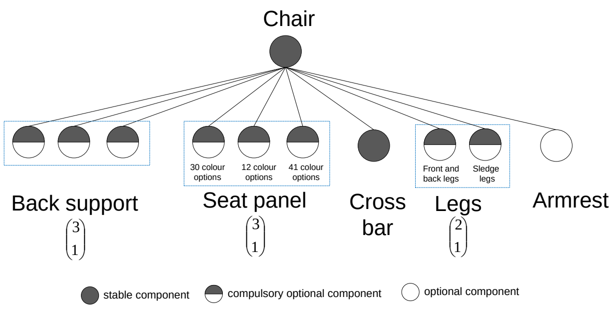


Fig. 2. Example with three types of initial components.

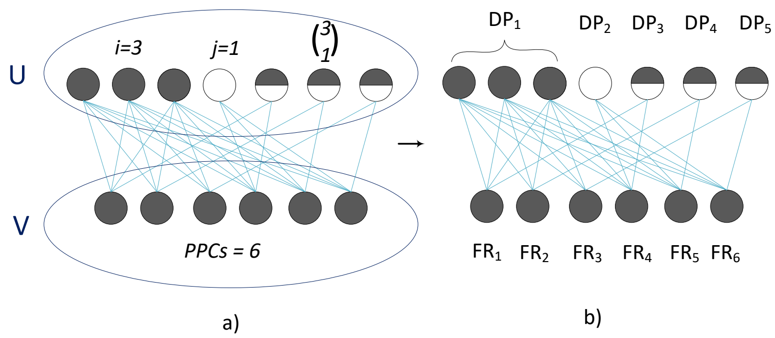
3.1.2. Graphical interpretation of relation between ICs and PPCs

Subsequently, it is useful to model the relation between ICs and PPCs by incomplete bipartite graphs (see an example in Figure 3a) with two independent sets of vertices U and V, where the set U consists of ICs and the set V contains from related PPCs. The set U in given case involves three stable components (*i=3*), one optional component (*j=1*), and three compulsory optional components with the individual selectivity rule $\binom{3}{1}$. Frequently, practitioners used to express product variety complexity by number of PPCs. However, the graphical models can be helpful to study product variety complexity in more detail way.

3.1.3. Transformation mechanism of scheme of relation between ICs and PPCs into a DM

The transit mechanism, which was partly outlined in our previous work [15], is based on substitution of set U consisting of ICs by DPs. Subsequently, elements of set V, i.e. PPCs will be replaced by FRs. Moreover, the transformation takes under consideration the fact that number of stable components does not impact on number of PPCs. Therefore, all stable components occurring as initial assembly inputs are represented only by one DP as shown in Figure 3b.

Finally, the bipartite graph represented by FR-DP relations can be easily transformed into a system design matrix shown in Figure 3c.



	DP_1	DP_2	DP_3	DP_4	DP_5
FR_1	X	0	X	0	0
FR_2	X	0	0	X	0
FR_3	X	0	0	0	X
FR_4	X	X	X	0	0
FR_5	X	X	0	X	0
FR_6	X	X	0	0	X

c)

Fig. 3. a) Two independent sets of vertices U and V; b) Relation between DPs and FRs; c) Final transformation of the relation between ICs and PPCs into DM.

Then, product variety complexity can be enumerated by using formula (2) as follows:

SDC= $\Sigma N_j \ln N_j$ = 6ln6 + 3ln3 + 2ln2 + 2ln2 + 2ln2 = 18,21 nats.

This transformation assumes that DPs considered as inputs components are determined by individual customer's specific needs through FRs of the product. Such defined FRs are directly included in selected product configuration. Then, the model of FR-DP relations depicted in Figure 3b is coherent with three of four domains of the design world [16] as shown in Figure 4.

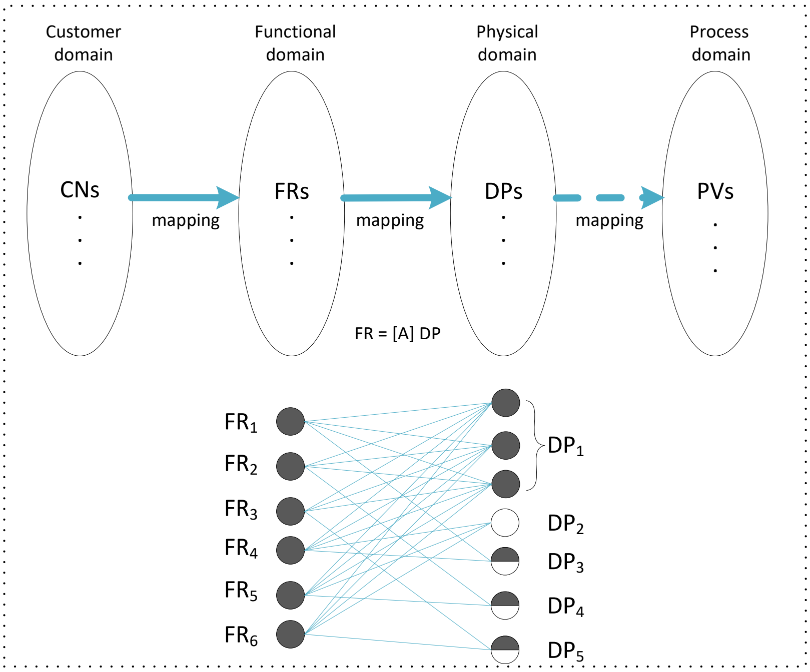


Fig. 4. The relation between ICs and PPCs in context of four domains of the design world.

3.2. Comparison of SDC against PPCs

In order to analyse possible differences between two possible product variety complexity measures, i.e., SDC and PPCs, two scenarios will be considered.

3.2.1. Scenario #1

In this scenario, structure of ICs is assumed as combination of two and more S components and one and more O components. Let's compare SDC and PPCs by using numbers of these ICs as described in Table 1.

Table 1. PPCs and SDC values for selected combinations of S and O components.

Number of S	Number of O	PPCs	SDC [nats]
2	1	2	1,39
	2	4	8,32
	3	8	33,27
3	1	2	1,39
	2	4	8,32
	3	8	33,27
4	1	2	1,39
	2	4	8,32
	3	8	33,27

As it can be seen from Table 1, number of *S* does not impact on PPCs and SDC values. This fact can be simply explained by transforming selected three combinations of ICs as shown in Figure 5.

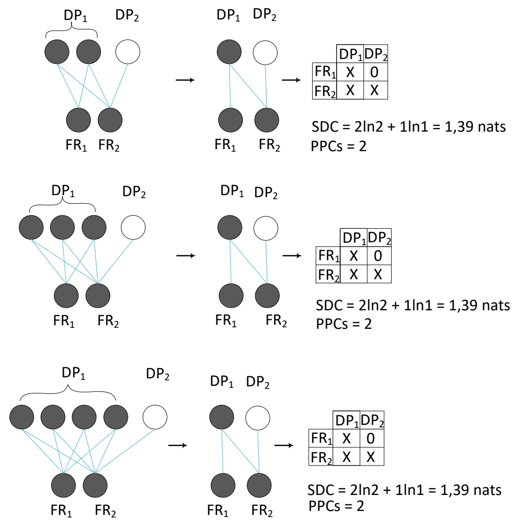


Fig. 5. Examples of combinations of ICs with different number of S.

In Figure 5, all stable components in original bipartite graphs are replaced only by one stable component. Then all three modified graphs are identical, which logically proves that SDC measure reflects this reality adequately.

3.2.2. Scenario #2

This scenario supposes several CO components with all possible individual selectivity rules. Let's use the example with five CO components and all possible individual selectivity rules, namely $\binom{5}{1}$, $\binom{5}{2}$, $\binom{5}{3}$ and $\binom{5}{4}$ as shown in Figure 6.

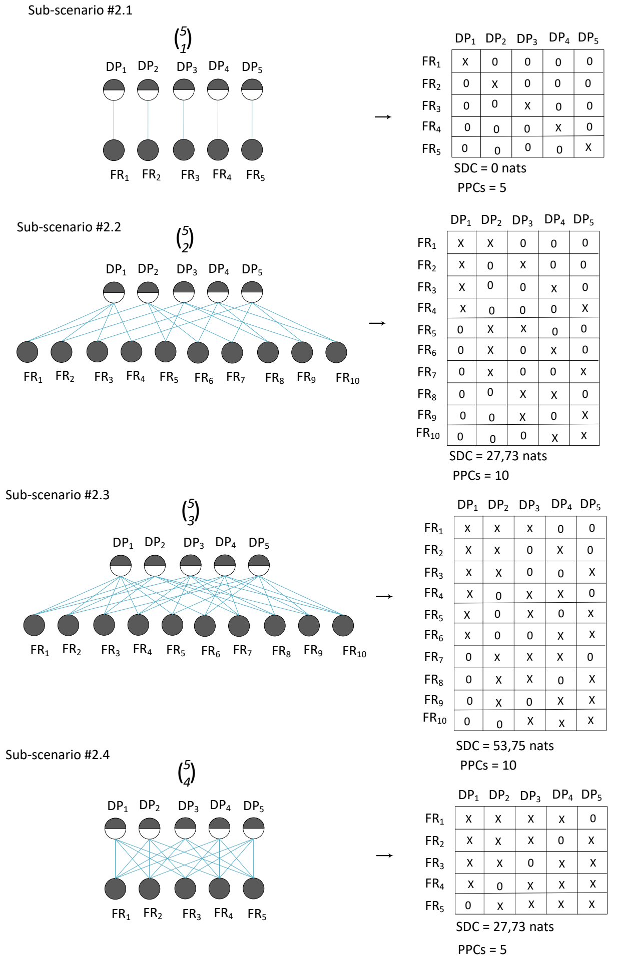


Fig. 6. An example of scenario with 5 CO components with all possible individual selectivity rules.

Then, it is easy to quantify SDC and PPCs values. Analogically, it could be possible to generate SDC and PPCs values for arbitrary number of CO components with all possible individual selectivity rules. SDC and PPCs values for such cases with five, six and seven CO components are graphically depicted in Figure 7.

As it can be seen from the Figure 7, PPCs and SDC values for a given number of CO components principally differ from each other. Especially, it can be pointed that number of PPCs for pairs of individual selectivity rules $\left[\binom{k}{1}, \binom{k}{k-1}\right]$ are equal, while SDC values for the same individual selectivity rules are different. Then, there is a question about which of the two indicators reflect the complexity in more realistic way. To find the answer, the example from Figure 6 will be used. By comparing two bipartite graphs for the rules $\binom{5}{1}$ and $\binom{5}{4}$, it is empirically evident that product variety complexity for the rule $\binom{5}{4}$ is higher than for the rule $\binom{5}{1}$. However, this practical view can be proved by formal verification.

For this purpose, the following definition for structural complexity will be employed. The complexity of a system scales with the number of its elements, the number of interactions between them, the complexities of the elements, and the complexities of the interactions (Gershenson 2002) [17].

By applying the previous definition for the two sub-scenarios *#2.1 and #2.4* from the Figure 6, the following results can be obtained.

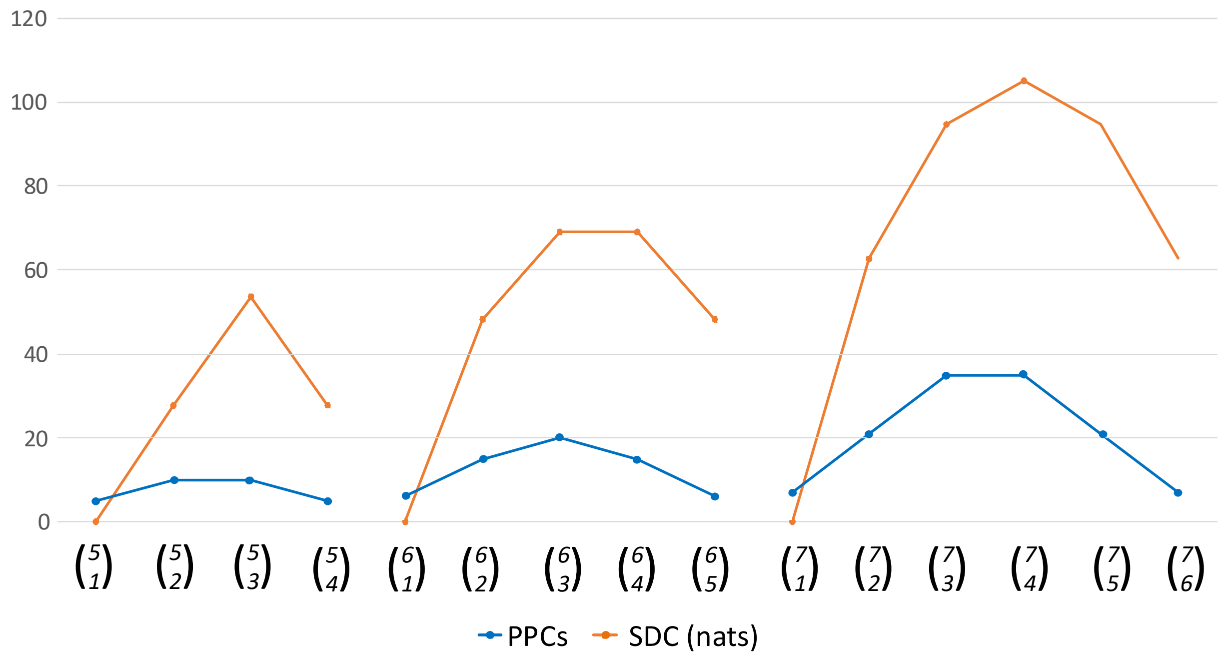


Fig. 7. Comparison of SDC and PPCs values.

Sub-scenario #2.1 with the rule $\binom{5}{1}$:

- number of nodes: 10,
- number of edges: 5,
- the complexity of the edges: one edge set
- the complexity of the nodes: one component set

Sub-scenario #2.4 with the rule $\binom{5}{4}$:

- number of nodes: 10,
- number of edges: 30,
- the complexity of the edges: one edge set
- the complexity of the nodes: four component set

Then, according to the previous complexity definition, it is proved that product variety complexity for the rule $\binom{5}{4}$ is higher than for the rule $\binom{5}{1}$.

By analogy it can be proved that product variety complexity for the rule $\binom{5}{3}$ is higher than for the rule $\binom{5}{2}$.

The same logic can be used to verify that SDC values better reflect product variety complexity than number of PPCs for different amount of compulsory optional components.

4 Adaptation of SDC to the process complexity measure

Similarly, as in the case of adaptation of SDC to measurement of product variety complexity, manufacturing process structure will be transformed into coupled design matrix. This transformation in subsequent subsection will be described.

4.1. Description of the process complexity measure

Prior to transformation of manufacturing process structure into DM, model of assembly process structure will be introduced.

4.1.1. Description of model of assembly process structure

Assembly type of operations are commonly interpreted using graph theory as convergent graphical models. Under convergent assembly structure we understand the chain where one process node has at most one successor, but has to have at least two predecessors. Our framework of assembly structures follows the work of Hu et al. [18], who outlined the way to model possible supply chain structures based on the number of original suppliers. An example of the model of assembly process structure is shown in Figure 8 a), where $i=1,2,\dots,m$ is the number of stable input assembly components; $O=1,2,\dots,p$ is the number of assembly operations, while O_0 represents final assembly operation.

4.1.2. Transformation mechanism of manufacturing process structures into a DM

In this transformation, equally as in Subsection 3.1.3, stable input components will be substituted by DPs and process nodes will be replaced by PVs (see Figure 8 b)). Subsequently, the convergent graph represented by DP-PV relations can be easily transformed into a system design matrix shown in Figure 8 c).

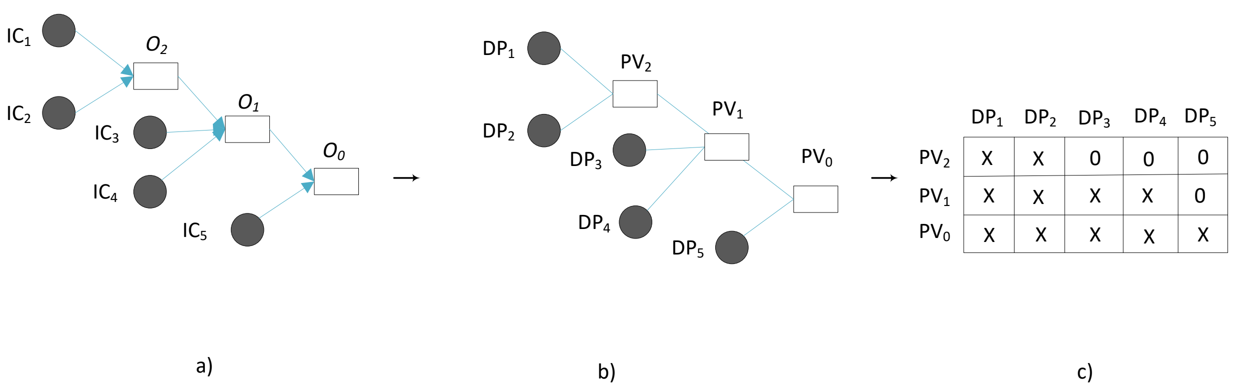


Fig. 8. a) Initial model of assembly process structure; b) Converted model of assembly process structure; c) Transformation of the converted process structure into DM.

Then, process complexity can be enumerated by using formula (2) as follows:

SDC= $\Sigma N_j \ln N_j = 3\ln3 + 3\ln3 + 2\ln2 + 2\ln2 + 1\ln1 =$ 9,36 nats.

In such graphs, input assembly components are mapped from physical domain to process domain as can be seen in Figure 9.

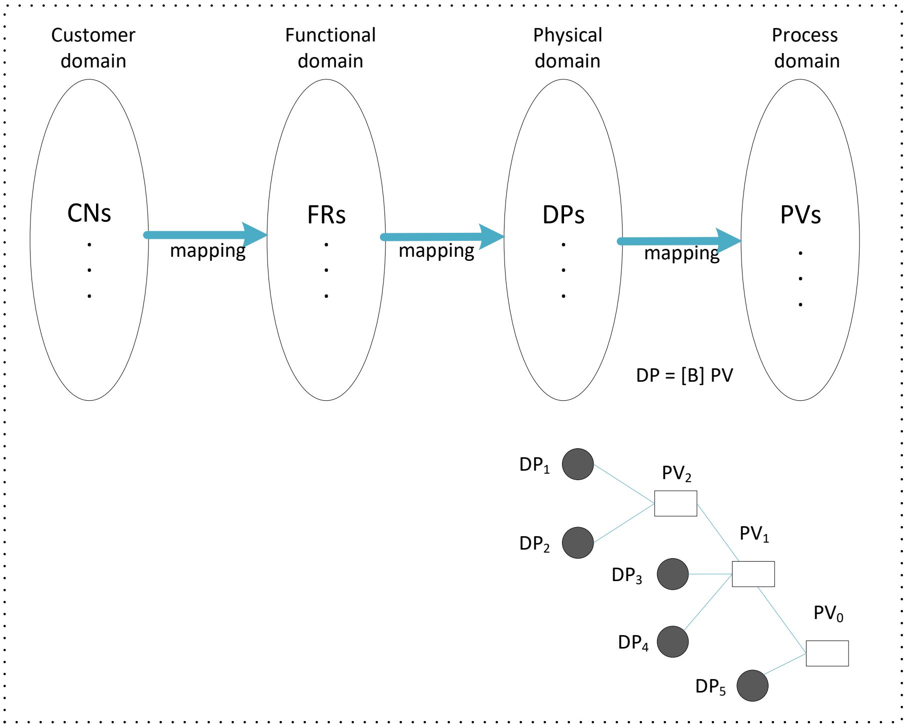


Fig. 9. The relation between input assembly components and process nodes in context of four domains of the design world.

4.2. Comparison of SDC against to concurrent measure

Competitive properties of the SDC measure can be found by its comparison with possible concurrent measures. Possible alternative complexity indicators were already mutually compared in previous studies [19, 20], where so called Index of vertex degree (Ivd) met optimality criteria for assessment of network complexity in the best way. Index of vertex degree has been introduced by Bonchev and Buck [21] and is expressed for Graph G consists of a set of *V* vertices, *{V}≡{v1, v2, … , vV},* by formula:

$$Ivd = - \Sigma^{v}{}_{i=1} \deg(v)_i \log_2 \deg(v)_i \qquad (3)$$

where *deg(v)* is the degree of vertex *v* in G.

Differences between these two process complexity measures, i.e., SDC and Ivd, will be analysed and evaluated through the following two computational experiments.

4.2.1. Description of computational experiment #1

The aim of the first experiment will be an investigation of complexity differences between assembly process structures resulting from product variants. For this purpose, real industrial case for assembly of chairs will be used. The chair contains of 7 assembly input components, namely, (1) back support, (2) cross bars, (3) front legs, (4) back legs, (5) sledge legs, (6) seat panel, and (7) arm rest. The feature product diagram describing the product structure is depicted in Figure 10.

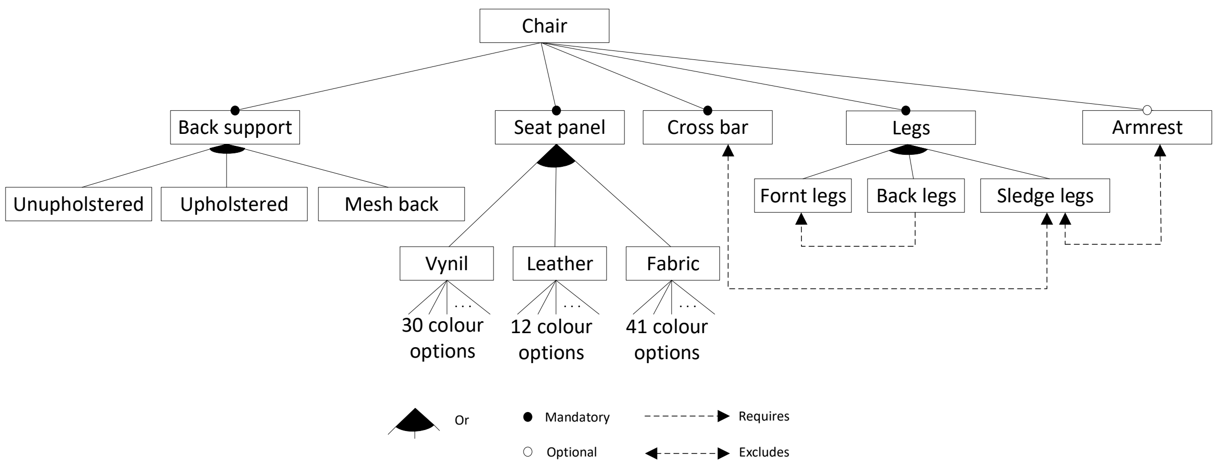


Fig. 10. Feature product diagram.

Customization of the chair includes colour and material variants of the seat panel, two modifications of the legs design, and three types of back support. Totally, there are 747 possible product variants. When material and colour varieties are omitted, there are three variants of product design and the same number of assembly process structures (see Figure 11).

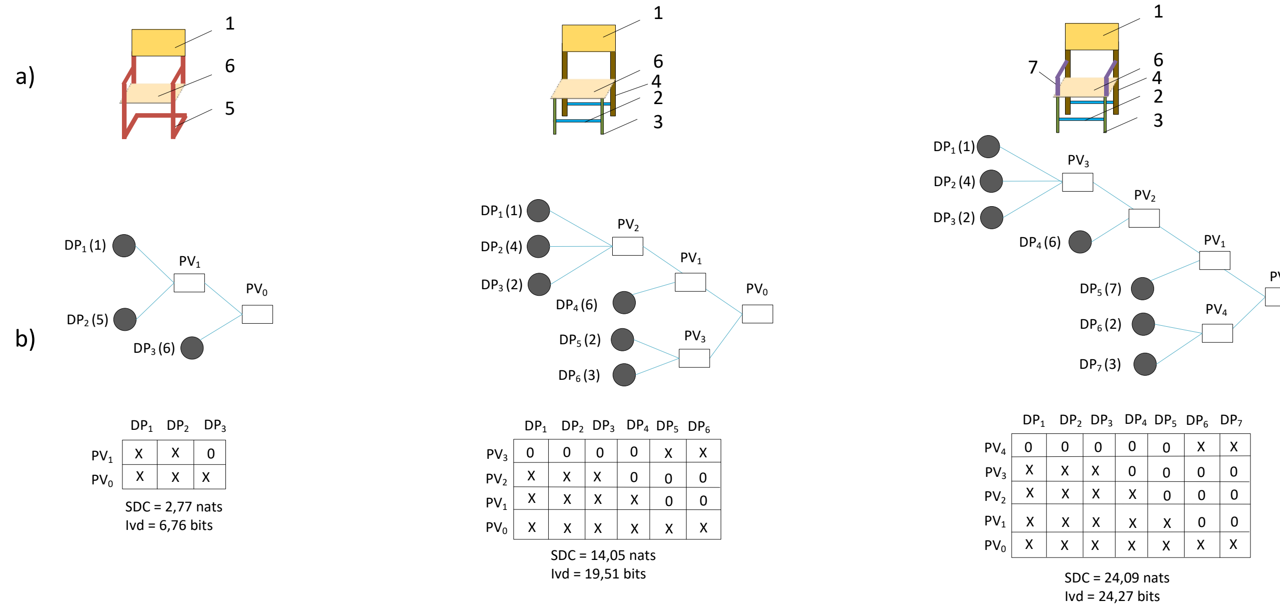


Fig. 11. a) Design variants of customized chair; b) Related process models of assembly operations with DMs.

For example, assembly process structure No. 1 consists of 2 operations. The first one includes assembly of back support with compact legs, and subsequently seat panel is added.

When applying two concurrent complexity measures, Ivd and SDC, for determination of topological complexity of the structures in Figure 11 b) one can see that complexity values of both indicators have the same tendency and provide similar results. It justified the applicability of the measurement method of SDC for intended purpose.

4.2.2. Description of computational experiment #2

The aim of the second experiment is to show differences in sensitivity between the two indicators. The experiment is based on the real assumption that each of the 3 assembly structures can be topologically modified by splitting or integrating process operations. For example, when number of initial assembly components is 6, then number of all possible process alternatives is 33. When we compared complexity of all possible theoretical process structures for 4, 5 and 6 initial input components we found that there are the same complexity values for different process structures by using Ivd indicator. For such process structure, we applied SDC indicator as shown in Figure 12.

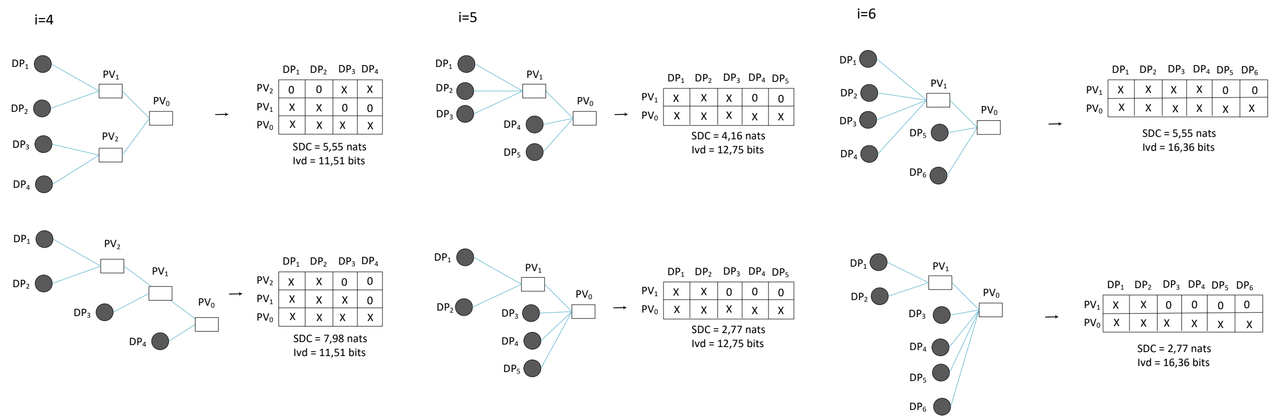


Fig. 12. An examples of process structures containing four, five and six assembly ICs with SDC and Ivd values.

As can be seen from obtained results in Figure 12, indicator SDC identifies different complexities between the pairs of the structures. It proves that SDC reflects the differences between these structures more sensitively.

5 Conclusion

The evaluations of the obtained results from computational experiments applied in the Section 3 and 4, indicate strong theoretical and promising practical potential of the two complexity techniques for the control and reduction of complexity in mass customization environment. The inherent properties of these two techniques for measuring observed complexity seem to be more suitable for given purpose than analysed concurrent indicators Ivd and PPCs.

Moreover, both indicators fit into the theoretical construct of axiomatic design not only from the viewpoint of partial dependencies between FRs-DPs and DPs-PVs, but there are clear mutual relations between all four domains of the design world as it is shown by Figures 4 and 9.

Finally, we would like to point out that our both proposed techniques for measuring observed product complexity and process complexity at the same time validated systems design complexity metric by Guenov [7] using fundamentals of Architectural Design and Axiomatic Design for comparison of alternatives. Even though this complexity metric does not include the information axiom of AD, it disposes of useful practical applications.

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